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THE IMPACT OF LEVERAGE REDUCTION ON THE EQUITY RISK LEVEL OF THE FIRM: an exploratory study of French firms

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ABSTRACT
Although it is admitted that financing decisions affect the equity risk of the firm, few studies have been dedicated to the analysis of the relationship between risk and leverage. In fact, to our knowledge, no study has addressed the question of whether leverage reduction has an effect on equity risk. This exploratory paper addresses this issue using data on French firms. The results of the study show that leverage reduction significantly reduces equity risk. However, an indirect test of the relative importance of this effect indicates that the reduction may not lead to shifts in risk classes. In other words, asset risk is the more important risk factor.

Key words: leverage reduction, equity risk, beta, risk class

RESUME
Bien qu'il soit admis que les décisions financières affectent le risque des fonds propres, il y a eu très peu d'études consacrées à l'analyse de cet impact. En effet, à notre connaissance, aucune étude ne s'est penchée sur la question de l'impact de la réduction de la dette sur le risque des fonds propres. Cette papier exploratoire tente de répondre à cette question avec des données de firmes françaises. Les résultats de l'étude montrent que le désendettement réduit significativement le risque des fonds propres. Cependant, un test indirect de l'importance de cet impact indique que la réduction n'induirait pas un changement de classe de risque. Autrement dit, le risque économique est le facteur le plus important.

Mots clés : désendettement, risque des fonds propres, bêta, classe de risque.

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It is generally admitted that financing decisions by firms affect their risk levels (in particular equity risk) and that systematic risk, as measured by beta inter alia, is the appropriate measure of risk to reckon with. Also, leverage is assumed to be positively correlated with the firm's equity risk. In other words, increases in leverage should, on the average, raise equity risk levels and, by corollary, decreases should lower them.

However, research studies, notably empirical, on the relationship between risk and the firm's financing decisions are very rare in the literature. Although it is undeniable that risk is an important factor in the decision processes of the firm. Moreover, the few related studies encountered in the literature concern the impact of leverage increases (Hamada, 1972; Ross, 1985; and Chung, 1989) or stock splits (Brennan, 1988; and Wiggins, 1992). Ross and Chung have both formally demonstrated an inverse relationship between leverage and asset risk. While the results of the empirical study by Hamada, indicates a positive relationship between equity risk and leverage.

We are unaware of any study that examines, either formally or empirically, the impact of leverage reductions on the equity risk of the firm. This scarcity of research may be due to the fact that, up to the late 1980's, the general tendency had been for firms to increase leverage especially during the relatively high inflation periods of the late seventies-to-mid eighties[1]. But during the latter part of the eighties, the interest rate-inflation-rate differential increased significantly in most industrialised countries. This differential hovered around an average of six hundred basis points in France and four hundred and fifty, for short-term loans, in the major industrial countries. This translated into high cost of debt financing[2]. In a context where "real" interest rates are prohibitively high, firms should normally be expected to reduce their leverage levels, ceteris paribus, especially if the economy is not performing satisfactorily, as has been the case in France recently (i.e. 1995-1996). A number of firms announced their intentions to significantly reduce the amount of debt in their capital structure during this period.

[1] This has been the case in France.
[2] see "Alternatives Economiques", p.44 - 47

It should be emphasised that independently of cost reductions, the lowering of leverage should, in accordance with the Miller and Modigliani (1958, 1963) propositions, have a negative impact on the risk of the firm's equity. It is, therefore, appropriate to examine whether these measures actually reduce risk, although this was not the explicit purpose of the announced leverage reductions.

The aim of this paper is to empirically explore the possibility that reductions in leverage levels negatively affect the risk of the firm's equity. It is organised in four sections. The first section defines and analyses the nature of the problem within the framework of the Miller and Modigliani propositions. In section two the empirical methodology, sampling and sample characteristics are presented and discussed. Section three analyses the results of the study. Finally section four concludes the paper.

1. Nature of the problem

Following Miller and Modigliani, the required rate of return on the equity of a levered firm, as compared to an otherwise identical but unlevered firm, may be given by the following expression:

$$r_i = r_a + (r_a - r_d)(1 - t_c)\frac{D}{E} + e_i \qquad [1]$$

Where:

r_a = the required risk adjusted rate of return on the unlevered firm's equity (i.e. the asset required rate of return)

r_d = the cost of the firm's debt (assumed to be risky)

D = the value of the firm's debt

E = the value of the firm's equity

τ_c = corporate tax rate

ε_i = random variable assumed to be iid.

Equation [1] implies that the market required rate of return on equity incorporates two categories of risk premium: economic and financial. The first category represents the risk

inherent in the economic activities of the firm, while the latter represents the risk of its financing decisions.

To separate the asset return factor from the leverage factor, equation [1] may be reformulated as follows:

$$r_i = r_a\left[1+(1-t_c)\frac{D}{E}\right] - r_d(1-t_c)\frac{D}{E} + e_i \qquad [2]$$

The above formulation enables us to express the covariance of the equity rate of return with the market portfolio rate of return as follows:

$$\begin{aligned} Cov(r_i, r_m) &= Cov\left\{\left[r_a\left(1+(1-t_c)\frac{D}{E}\right) - r_d(1-t_c)\frac{D}{E}\right]; r_m\right\} \\ &= \left[1+(1-t_c)\frac{D}{E}\right]\mathrm{cov}(r_a, r_m) - (1-t_c)\frac{D}{E}\mathrm{cov}(r_d, r_m) \end{aligned} \qquad [3]$$

According to equation [3], the covariance between the required rate of return on the equity of a firm and that on the market portfolio is a function of three factors:

i) the impact of market movements on the economic activities of the firm

ii) the impact of market movements on the value of the firm's debts

iii) the level of the debt-to-equity ratio of the firm that reflects the financial risk level assumed by the firm through its financing decisions.

Consequently, the systematic risk of the firm's equity may be obtained as follows:

$$b_i = b_a\left[1+(1-t_c)\frac{D}{E}\right] - b_d(1-t_c)\frac{D}{E} \qquad [4]$$

β_i = firm's equity systematic risk

β_a = firm's asset (economic) systematic risk

β_d = firm's leverage systematic risk

From equation [4], it can be shown that a reduction in the firm's leverage level may have positive, negative or no effect on its equity risk. Indeed, any change in the capital structure

may affect not only the debt-to equity ratio, but also the asset and debt risk levels, as shown by the following differential expression[3]:

$$\frac{db_i}{dD}\Big|_{dD<0} = -\left[\frac{db_a}{dD}\left(1+(1-t_c)\frac{D}{E}\right) - \frac{db_d}{dD}(1-t_c)\frac{D}{E} + (1-t_c)\frac{b_a}{E} - (1-t_c)\frac{b_d}{E} - (1-t_c)\frac{dE}{dD}\frac{D}{E^2}b_a + (1-t_c)\frac{dE}{dD}\frac{D}{E^2}b_d\right]$$

$$= -\frac{db_a}{dD} - \left(\frac{db_a}{dD} - \frac{db_d}{dD}\right)(1-t_c)\frac{D}{E} - (b_a - b_d)\frac{(1-t_c)}{E} + (b_a - b_d)(1-t_c)\frac{dE}{dD}\frac{D}{E^2}$$

By rearranging the terms, the above expression can be simplified as follows:

$$\frac{db_i}{dD}\Big|_{dD<0} = \frac{1}{E}\left[(b_a - b_d)(1-t_c)\frac{D\frac{dE}{dD} - E}{E} - \frac{db_a}{dD}E - \left(\frac{db_a}{dD} - \frac{db_d}{dD}(1-t_c)\right)D\right] \qquad [5]$$

It may be noted that the first term in the above expression should always be negative, because β_a is, ex ante, greater than β_d and $dE/dD \leq 0$. However, the sign of the second and third terms will depend on the way the reduction in the firm's leverage level is financed. In effect, it may be financed in three ways:

i) the firm may use its internal resources (cash and other assets)

ii) it may offer to exchange its debt for equity

iii) it may issue new shares and use the proceeds.

If the firm uses internal resources to finance the change in policy, the impact of such a decision will depend on the risk of the assets disposed of. If they are amongst the riskiest assets in its portfolio, the impact of such a decision will be to lower the levels of both the asset and debt risks. Formally, this implies that:

$$\frac{db_a}{dD} > 0 \text{ and } \frac{db_d}{dD} > 0$$

Hence, the overall incidence, of such a decision, on the firm's equity risk should be negative; that is the firm's equity risk should fall. This result becomes clear by noting that, in this case, $dE/dD = 0$ and that equation [5] consequently becomes:

[3] The negative sign implies reduction in the amount of debt

$$\frac{db_i}{dD} = -\frac{1}{E}\left[(b_a - b_d)(1 - t_c) + \frac{db_a}{dD}E + \left(\frac{db_a}{dD} - \frac{db_d}{dD}(1 - t_c)\right)D\right] \qquad [5a]$$

On the other hand, if the assets involved are amongst the least risky, the decision should increase both risk measures; viz:

$$\frac{db_a}{dD} < 0 \text{ and } \frac{db_d}{dD} < 0$$

The fact that the decision increases the asset risk in this case is intuitively clear. The implied impact on the risk of debt should also be clear since the remaining outstanding debts are, thereby, covered by riskier portfolio of assets. However, the overall impact of such a "strategy" on equity risk will depend on the magnitude of the difference between the value of the first term, in equation [5], and the sum of the values of the second and third terms. Indeed the resultant relationship will be as given below:

$$\frac{db_i}{dD} = -\frac{1}{E}\left[(b_a - b_d)(1 - t_c) - \frac{db_a}{dD}E - \left(\frac{db_a}{dD} - \frac{db_d}{dD}(1 - t_c)\right)D\right] \qquad [5b]$$

It is evident from equation [5b], that if the firm uses the least risky assets in its portfolio to reduce its leverage, the overall impact could be positive, nil or negative.

In the case where the reduction in leverage is financed through increases in equity (new issues or debt-for-equity swap), the asset risk will not be affected since the firm's portfolio of assets will not be modified. But the risk of debt will be lowered; i.e.:

$$\frac{db_a}{dD} = 0 \text{ and } \frac{db_d}{dD} > 0$$

This result is due to the fact that, although the portfolio of assets is not affected, the remaining outstanding debts will be proportionately better secured. The overall impact on equity risk should, therefore, be downward, as could be deduced from the following relationship[4]

[4] Note that in this case dE/dD is negative.

$$\frac{db_i}{dD} = \frac{1}{E}\left[(b_a - b_d)(1 - t_c)\frac{D\frac{dE}{dD} - E}{E} + \frac{db_d}{dD}(1 - t_c)D\right] \qquad [5c]$$

In the light of the above analyses, it can be argued that, ex ante, the sign of the impact of a leverage-reducing financial decision on the risk of equity cannot be determined. It is a function of the way the decision is implemented. But, within the framework of the Myers-Majluf "pecking order" propositions, firms are likely to prefer using internal resources to finance leverage reduction decisions. Moreover, in times of difficult economic conditions, such as the late 1980s to early1990s period, they would prefer divesting the riskiest assets. As such, the overall impact on equity risk, in a cross sectional sample of firms, should be negative.

It is on the basis of the above argument that we hypothesise that, in a period of high interest-rate-inflation-rate differential such as that experienced in the late eighties to early nineties, the equity risks of a randomly selected sample of firms, should on the average fall as a result of reductions in their debt levels.

2. Methodology and Data collection

2.1. Methodology

To test the hypothesis that leverage reduction may have a negative impact on the risk of equity, the following steps are followed.

First two sets of beta, using the market model (i.e. OLS regression technique), are estimated: one before the debt reduction was observed and the other afterwards. Since the exact date on which the debt reduction was realised cannot be ascertained, the one-year period separating the publication of the 1988 and 1989 financial statements, was considered as the event or "announcement" window. Hence the beta estimates do not include the equity return series within this window. The consistency and accuracy of OLS estimates have been the subject of a number of research and debates in the financial literature (Black, Johnson and Scholes, 1972, Fama and French, 1996, Isakov, 1996). Yet no definitive answer has been provided

with regards to the "appropriateness" of the technique. However, the differencing approach adopted here should attenuate the measurement errors associated with the OLS technique; especially if these errors are consistent over time.

Secondly, the magnitude of the differences between the two sets is tested using two related techniques:

i) the first is based on the difference between the averages of the beta

ii) the second is based on the average of the beta differences.

The first test assumes that beta variances in each period are not equal, while the second assumes equality of the variances.

The empirical models used are presented below:

i) Test of the significance of the difference between the beta averages:

$$\frac{E(b_t) - E(b_{t+j})}{\sqrt{\frac{s_t^2}{n_t} + \frac{s_{t+j}^2}{n_{t+j}}}} > 0 \qquad [6a]$$

β_t = the risk measure before the leverage reduction

β_{t+j} = the risk measure after the leverage reduction

ii) Test of the significance of the average of the beta differences

$$\frac{E\left(b_{t,i} - b_{t+j,i}\right)}{\sqrt{\frac{s_\Delta^2}{n}}} \qquad [6b]$$

s_Δ^2 = variance of the beta differences

The above tests are complemented with a test based on Spearman's rank correlation coefficient. The aim of the latter is to indirectly examine the relative importance of leverage in equity risk. The hypothesis being tested here is that leverage policy may not significantly

change the firm's risk class because the latter is predominantly determined by the nature of its economic activities. The significance of the rank correlation was tested using the following relationship[5]:

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \qquad [7]$$

t is assumed to be t-distributed with (n-2) degrees of freedom.

2.1. Data collection and their characteristics

First, information was collected on the consolidated financial statements of all the companies listed on the Paris Bourse whose debt levels (in monetary terms) had fallen between 1988 and 1989. The source of the data are the various issues of the "Annuaire Défossés" a compendium of the publicly listed companies on the Paris stock exchange[6]. In all 160 firms fell into this category. The sample was limited to the two principal exchanges of the Paris Bourse (Marché Comptant[7] and Marché à Réglement Mensuel[8]). This limitation was aimed at reducing measurement problems associated with thin trading that characterise the other segments (Second Marché and Hors Cote) which happen to be less liquid.

Data on stock market returns were obtained from the SBF-AFFI database. In order to minimise problems relating to the stability of beta and rates of return auto-correlation, weekly rates, rather than monthly or daily rates, were collected. To be retained in the sample, a firm must have at least 52 series of rates of return during each of the periods before the end of financial year of 1988 and especially after that of 1989[9]. This corresponds to a little over six months in each period. Rates of return between 1988 and 1989 were not included in order not to include market reactions to the reductions in the estimates. This requirement reduced the sample to 152 firms. This sample is henceforth referred to as the "total sample".

[5] see Johnston (1972), p.36
[6] Banks and insurance companies were excluded from the sample.
[7] The spot transactions compartment of the Paris exchange.
[8] The one-month forward transactions compartment.
[9] The length of the estimation period was constrained by data availability

The distribution of the sample of the 160 firms is presented below:

TABLE 1.
Sample distribution by level of debt reduction

Debt level reduction (%)	Number of firms	Proportion (%)
1 - 10	54	34
11 - 20	25	16
21 - 30	27	17
31 - 40	19	12
41 - 50	8	5
51 - 60	5	3
61 - 70	1	1
71 - 80	3	2
81 - 90	5	3
91 - 100	13	8

From the above table, it appears that about 80% of the firms have reduced their debt levels by less than 20%. Generally, the higher the proportion of the reduction, the lower the number of firms involved. It is, however, noteworthy that over 13% of the firms could be considered as virtually eliminating their debt (i.e. a reduction of between 71 and 100%).

Table 2 below presents the sample distribution by industrial sector as provided in the "Annuaire Défossé". This information is only indicative as the classification used is the first level and it is, therefore, crude. For instance, the financial sector is made up of holding companies, mutual funds and portfolio management firms. However it gives an idea of the "dispersion" of the firms that reduced their leverage during the period under study.

TABLE 2. Sample distribution by sector

	Sample		Population
Sector	Observation	Proportion	Proportion*
Primary products	19	**12%**	**9%**
Construction	9	**6%**	**7%**
Equipment	19	**12%**	**13%**
Durable consumer products	13	**8%**	**6%**
Non durables	7	**4%**	**12%**
Food products	13	**8%**	**7%**
Services	53	**33%**	**27%**
Financials	23	**14%**	**14%**
Foreign based	4	**3%**	**6%**
Total	160	100%	100%

*For a total of 750 firms

The distribution of the sample by industrial appears to be fairly similar to that of the population. Although the estimated chi-squared of 16.03 happens to be significant and a cursory analysis shows that the services and primary products sectors seem to be "over" represented. On the other hand, the non-durable consumer products and the foreign bases[10] firms appear to be under represented .

3. Results of the empirical tests

For the purpose of the empirical study, the full sample was divided into sub samples as follows:

Sub sample A: 131 firms which have reduced their leverage by at least 5%

Sub sample B:107 firms which have reduced their leverage by at least 10%

Sub sample C: 89 firms which have reduced their leverage by at least 15%

Sub sample D: 80 firms which have reduced their leverage by at least 20%

Sub sample E: 71 firms which have reduced their leverage by at least 25%

The idea behind this sub division is to verify whether the impact of leverage reduction is, as one would normally expect, a function of its relative importance. We discuss the results of the various tests in the following sections. We start first with a discussion of a summary of the descriptive statistics before discussing the results of our tests.

3.1. Summary of the estimation results

An interesting feature that could be noted in table three is the relative closeness of the estimates regardless of the period and the sample size. The maximum betas are all close to 2, the averages are well under one (i.e. the sample not does appear to reflect the "market" index) and minimum betas are all negative. The latter should be expected given the length of the estimation periods. It is also interesting to note that the variances, after the leverage reduction are invariably higher than those before the reduction. This increased dispersion may reflect probable differential firm-specific impact of the leverage reduction.

TABLE 3. Descriptive summary statistics of the beta estimates

	Total sample	Sub sample A	Sub sample B	Sub sample C	Sub sample D	Sub sample E
Statistics Before the leverage reduction						
Maximum	2,01	2,01	1,91	1,91	1,91	1,91
Average	0,72	0,74	0,73	0,69	0,69	0,70
Minimum	-0,48	-0,48	-0,12	-0,12	-0,12	-0,12
Variance	0,21	0,21	0,20	0,20	0,21	0,21
Sample size	152	131	107	89	80	71
Statistics After the leverage reduction						
Maximum	2,05	2,05	1,97	1,97	1,97	1,97
Average	0,59	0,61	0,57	0,58	0,58	0,60
Minimum	-0,49	-0,49	-0,49	-0,49	-0,49	-0,14
Variance	0,27	0,28	0,27	0,29	0,31	0,30
Sample size	152	131	107	89	80	71

Sub sample A = firms which reduced their leverage by at least 5%, Sub sample B = firms which reduced their leverage by at least 10%, Sub sample B = firms which reduced their leverage by at least 15%, Sub sample B = firms which reduced their leverage by at least 20%, Sub sample B = firms which reduced their leverage by at least 25%

3. 2. The empirical results of the tests of the beta differences

The table below presents the results of our various tests on the beta differences and of the rank correlation estimations. The first two columns are the outcome of the test based on the assumption that the variances in the two periods are different. Hence, to test for the beta differences, we estimated the average betas and their cross sectional variances for both periods, which we then used to test for the significance of the differences. The next two columns are estimates based on the assumption that the variances in both periods are equal. Therefore, we took the beta differences per firm and calculated the average and variance of these differences for our tests.

[10] These are firms, in various sectors whose activities are principally abroad.

TABLE 4. The empirical results of the tests of the beta differences

	Equal variance		Unequal variance		Rank correlation	
	Difference	t-student	Difference	t-student	coefficient	t-student
Total sample	-0,13	-3,61	-0,13	-2,28	0,61	9,33
Sub sample A	-0,13	-3,24	-0,13	-2,07	0,61	8,73
Sub sample B	-0,15	-3,75	-0,15	-2,30	0,64	8,63
Sub sample C	-0,11	-2,62	-0,11	-1,47	0,68	8,66
Sub sample D	-0,10	-2,41	-0,10	-1,30	0,71	8,91
Sub sample E	-0,09	-2,02	-0,09	-1,10	0,71	8,34

Sub sample A = firms which reduced their leverage by at least 5%, Sub sample B = firms which reduced their leverage by at least 10%, Sub sample B = firms which reduced their leverage by at least 15%, Sub sample B = firms which reduced their leverage by at least 20%, Sub sample B = firms which reduced their leverage by at least 25%

All the differences are negative, implying that leverage reduction, given a random sample and in periods of high interest-rate- inflation-rate differential, has an effect on the firm's equity risk. However, the significance of this impact does not appear to be "robust" with respect to the statistical techniques adopted. Whereas, the equal variance difference test would suggest that, whatever the degree of the reduction, the impact is statistically significant. This does not seem to be the case with the unequal variance test. The outcome in this case is counterintuitive in the sense that the significance of the reduction appears to be "curvilinear". The t-Student increases with leverage reduction up to about 10% (sub sample B) and declines afterwards. This is also the general picture with the equal-variance test even though all the tests in this case are significant at the traditional confidence levels. Nevertheless, one can reasonably argue that, overall, leverage reduction has a significant and negative impact on the firm's equity risk.

As mentioned earlier, another issue that we attempt to address in this paper is whether the leverage induced fall in equity risk is so important as to cause shifts in risk classes. We use the Spearman's rank correlation coefficient as an indirect means of addressing this issue. The hypothesis is that if there are no shifts in risk classes then the correlation coefficients will be statistically significant. This hypothesis is confirmed by the results in the above table, since the correlation coefficients are statistically significant (at the usual confidence levels) regardless of the degree of leverage reduction. This result, therefore, implies that, although leverage reduction is a factor to reckon with in the analysis of equity risk, the more significant factor is the asset risk.

4. Conclusion

This paper studied the relationship between leverage reduction and the firm's equity risk. It identified three means by which a firm could reduce it leverage and their implications for equity risk. The results of the empirical tests showed that leverage reductions do reduce equity risk, but that the major part of this risk has more to do with the firm's economic activities.

We may infer from the empirical results that the firm's financial policy is relevant for the equity risk of the firm. A firm that wishes to reduce the risk of its equity could do so by reducing it leverage. In situations of economic downturn, as was the case in the late eighties-to mid nineties, the combination of increased probability of distress and difficulty to benefit from interest tax shields should incite firms to reduce their leverage in order to lower their risks.

To conclude we would like to emphasise the fact that this is an exploratory study which essentially should raise questions with respect to leverage reduction. For instance, it may be interesting to investigate the existence of a "pecking order" in the way firms reduce their debt. It may also be useful to determine the information content and the nature of market reaction to announcements of leverage reduction. It may also be useful to verify the relevance to equity risk of the source of financing leverage reduction.

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